Oxford Cambridge and RSA

Tuesday 21 June 2022 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Insert

Time allowed: 2 hours

9122294576

INSTRUCTIONS
- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

INFORMATION
- This Insert contains the article for Section B.
- This document has **4** pages.

© OCR 2022 [603/1002/9]
DC (LK/SG) 300623/3
OCR is an exempt Charity

Approximating the sine function

Small angles

For a small angle x radians, the approximation $\sin x \approx x$ is valid. The curve $y = \sin x$ and the straight line $y = x$ are shown in **Fig. C1.1**. **Fig. C1.2** shows the curve $y = x - \sin x$. Inspection of the graphs suggests that x is a reasonable approximation for $\sin x$ for $-0.5 \leqslant x \leqslant 0.5$ and also that $y = x$ has the same gradient as $y = \sin x$ when $x = 0$.

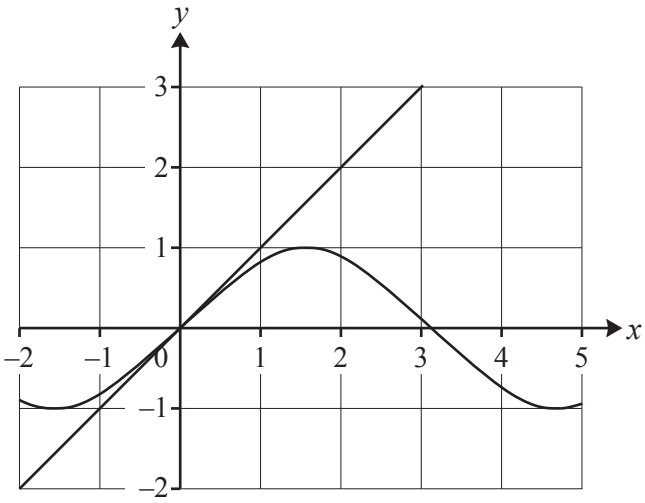


Fig. C1.1

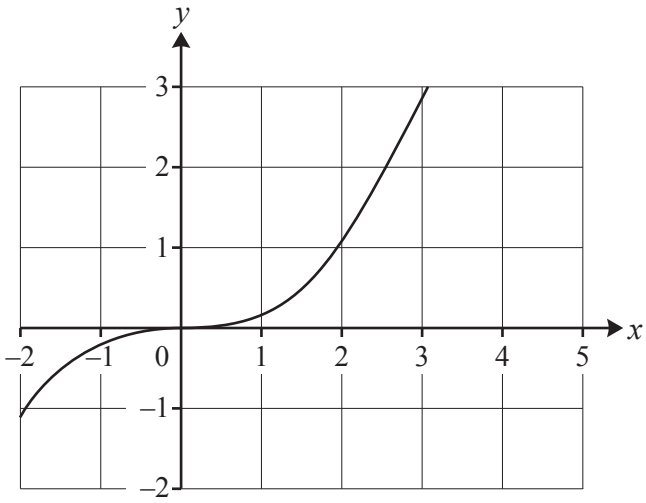


Fig. C1.2

Calculating sin *x*

Trigonometric functions, including $\sin x$, are widely used so it is useful to be able to calculate the value of the sine of any angle accurately and quickly. This is easily done nowadays using a calculator but this was not possible in the past. The linear function, $y = x$, is only a reasonable approximation for $y = \sin x$ for values of x close to zero. Perhaps using a higher degree polynomial would give a reasonable approximation for a wider range of values of x.

Fig. C2.1 shows the curve $y = \sin x$ and the quadratic curve which goes through the points (0, 0), $\left(\frac{\pi}{2}, 1\right)$ and $(\pi, 0)$. The equation of this curve is $y = \frac{4x(\pi - x)}{\pi^2}$. **Fig. C2.2** shows the curve $y = \frac{4x(\pi - x)}{\pi^2} - \sin x$.

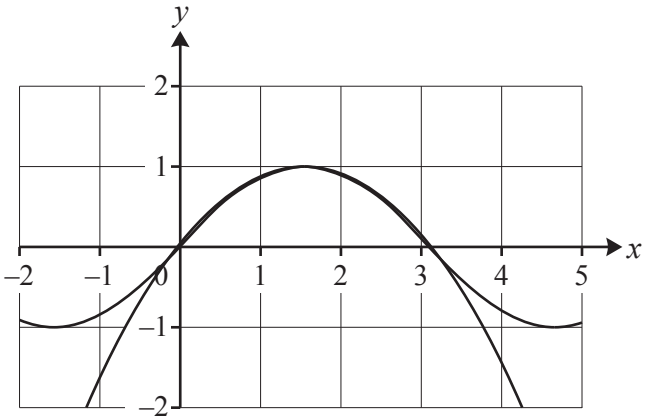


Fig. C2.1

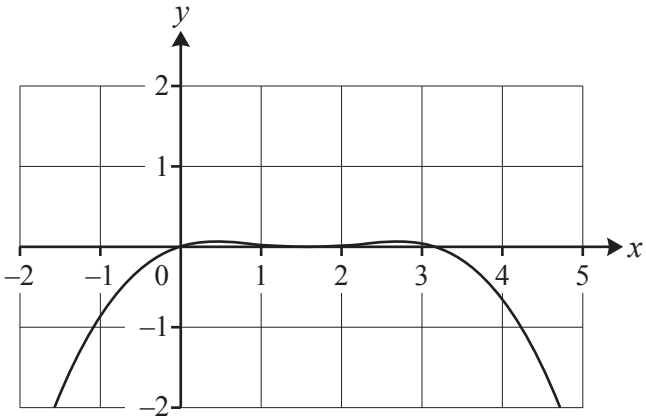


Fig. C2.2

© OCR 2022

The quadratic function seems to be a reasonably good approximation for $\sin x$ in the interval $0 \leqslant x \leqslant \pi$. However, calculating percentage errors for selected values of x shows that the percentage errors made by using the quadratic function as an approximation to $\sin x$ are quite high for values of x close to zero or π.

The spreadsheet in **Fig. C3** shows values of x in column A, with the corresponding values of $\sin x$ and the quadratic function $\frac{4x(\pi - x)}{\pi^2}$ in columns B and C. Columns D and E show the percentage errors in using x and the quadratic as approximations for $\sin x$.

	A	B	C	D	E
1	x	sin(x)	quadratic	% error for x	% error for quadratic
2	0	0	0		
3	0.1	0.099833	0.123271	0.166861	23.476799
4	0.2	0.198669	0.238437	0.669791	20.016773
5	0.3	0.295520	0.345496	1.515901	16.911206
6	0.4	0.389418	0.444450	2.717298	14.131825

Fig. C3

A better approximation

The approximation $\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ was discovered by an Indian mathematician named Bhaskara in the 7th century. It is not known how Bhaskara derived the formula but it can be seen that the curve $y = \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ is symmetrical about $x = \frac{\pi}{2}$ and goes through the points (0, 0), $\left(\frac{\pi}{2}, 1\right)$ and $(\pi, 0)$. **Fig. C4** shows the curves $y = \sin x$ and $y = \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$. Radians were not in use until the 18th century; Bhaskara gave the formula for an angle θ degrees as $\sin\theta \approx \frac{4\theta(180 - \theta)}{40\,500 - \theta(180 - \theta)}$.

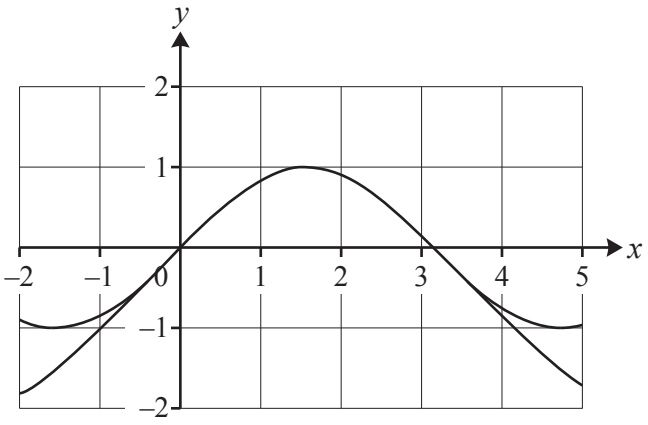


Fig. C4

The percentage error in approximating $\sin x$ by $\frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$ is less than 2% throughout the interval $0 \leqslant x \leqslant \pi$. The Bhaskara approximation for $\sin x$ can be used to derive the following approximation for $\cos x$; $\cos x \approx \frac{\pi^2 - 4x^2}{\pi^2 + x^2}$.

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2022